

# 1 Continuous powerlaw with minimum cut-off

Exponent  $\tau > 1$ .

## 1.1 Also maximum cut-off

Let  $m$  be the minimum,  $M$  be the maximum.

PDF:

$$f(x) = \frac{\tau - 1}{m^{-(\tau-1)} - M^{-(\tau-1)}} x^{-\tau}$$

CDF:

$$F(x) = \frac{m^{-(\tau-1)} - x^{-(\tau-1)}}{m^{-(\tau-1)} - M^{-(\tau-1)}}$$

Inverse:

$$F^{-1}(y) = \left( (1 - y)m^{-(\tau-1)} + yM^{-(\tau-1)} \right)^{\frac{-1}{\tau-1}}$$

i.e. linear interpolate between  $M^{-(\tau-1)} < m^{-(\tau-1)}$ .

$F^{-1}(0) = m$  and  $F^{-1}(1) = M$ .

For  $m = 1$  and  $M$  steps of interpolation:

$$F^{-1}(1/M) = \left( 1 - M^{-1} + M^{-\tau} \right)^{\frac{-1}{\tau-1}}$$

$$F^{-1}(1 - 1/M) = \left( M^{-1} + M^{-(\tau-1)} - M^{-\tau} \right)^{\frac{-1}{\tau-1}}$$

## 1.2 No maximum cut-off

For  $M = \infty$  we have:

PDF:

$$f(x) = \frac{\tau - 1}{m^{-(\tau-1)}} x^{-\tau} = \frac{\tau - 1}{m} \left( \frac{x}{m} \right)^{-\tau}$$

CDF:

$$F(x) = \frac{m^{-(\tau-1)} - x^{-(\tau-1)}}{m^{-(\tau-1)}} = 1 - \left( \frac{x}{m} \right)^{-(\tau-1)}$$

Inverse:

$$F^{-1}(y) = \left( (1 - y)m^{-(\tau-1)} \right)^{\frac{-1}{\tau-1}} = m (1 - y)^{\frac{-1}{\tau-1}}$$

For interpolation:

$$F^{-1}(0) = m$$

$$F^{-1}(1 - \frac{1}{n}) = m \cdot n^{\frac{1}{\tau-1}}$$